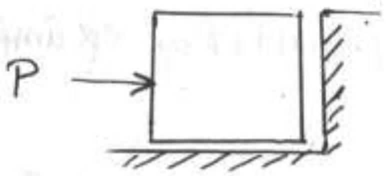
Engineering Mechanics

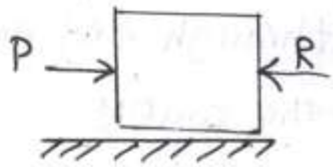
Introduction

Mechanics is that branch of Phisical science which considers the motion of bodies, with rest as a special case of motion.

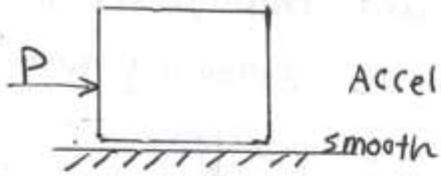
The external effect of a force on a body is either to accelerate the body or to develop resisting forces (reactions) on the body

when the force system acting on a body is balanced, the system has no external effect on a body, the body is in equilibrium, and the problem is one of statics. when the force system has a resultant different from zero, the body will be accelerated, and the problem is one of dynamics





Equilibrium



Acceleration of the body (Not equilibrium)

Rigid body: A body in which all particles remain at fixed distances from each other is called a rigid body. No real body is absolutely regid, but in many cases the changes in shape of the body have a negligible effect upon the acceleration produced by a force system or upon the reactions required to maintain equilibrium. whenever the changes in distance between the particles of a body can be neglected, the body is assumed to be rigid.

Scalar & Vector Quantities

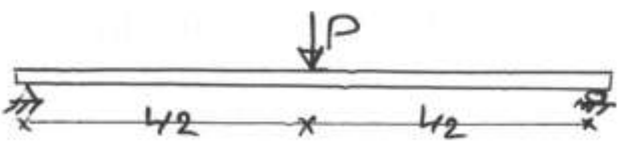
Ascalar quantity: is one which has only magnitude such as mass volume & time

Avector quantity: is one which involves both magnitude and direction, so that it can be represented by a directed line segment and which conforms to the parallelogram law of addition such as velocity, Acceleration & force

Vector Quantity → Free Vector
Vector Quantity → Localized Vector

A Free vector is one with a specified slope and sense but no acting through any particular point, for example the moment of the couple

Alocalized vector act through a particular point, for example the force (P) on the following beam



Force:

The characteristics of a force, which describe its external effect on a rigid body, are :

(a) its magnitude,
(b) its direction (sense and slope), and
(c) the location of any point on its line of action.

Figure (1-1) shows a convenient way of indicating all the characteristics of a force in a plane.

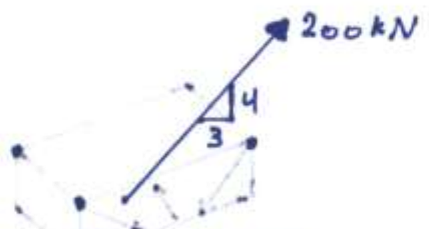
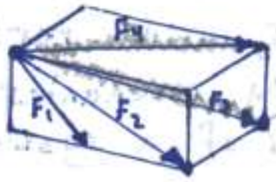


Fig. (1-1)

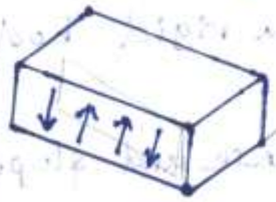
When several forces act in a given situation, they are called a system of forces or a force system. Force systems can be classified according to the arrangement of the lines of action of the forces of the system as follows :

Collinear. All forces of the system have a common line of action.

Concurrent, Coplanar. The action lines of all the forces of the system are in the same plane and intersect at a common point



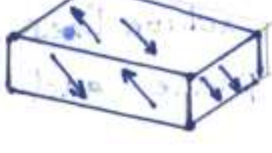
<u>Parallel, coplanar.</u> The action lines of all the forces of the system are parallel and lie in the same plane.



<u>Non concurrent, nonparallel, coplanar.</u> The action lines of all the forces of the system are in the same plane, but they are not parallel and they do not all intersect at a common point.

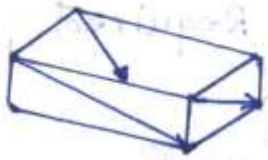
<u>Concurrent, noncoplanar.</u> The action lines of all the forces of the system intersect at a common point, but they are not all in one plane.

* <u>Parallel, non coplanar.</u> The action lines of all the forces of the system are parallel, but they are not all in the same plane.



د. سعد

(7) Non concurrent, nonparallel, noncoplanar. The action lines of the forces of the system do not all intersect at a common point, they are not all parallel, and they do not all lie in the same plane.



The resultant of a force system is the simplest force system which can replace the original system without changing its external effect on a rigid body. The resultant of a force system can be a single force, a pair of parallel forces having the same magnitudes but opposite senses (called a couple), or a force and a couple. If the resultant is a force and a couple, the force will not be parallel to the plane containing the couple.

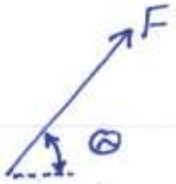
كل قوة لها قيمة واتجاه ، اذا كانت القوة في المستوى (In plane)
او في الفضاء (In space) يمكن تحليلها الى مركبات (components)
متعامدة . والعكس صحيح اذا تم ايجاد المركبات (components)
يمكن ايجاد القوة الاصلية (المحصلة resultant).

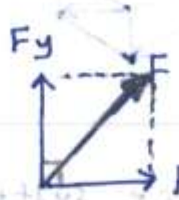
In-plane

مستوي

1 Resolve force into two rectangular components.

a) Given F, θ. Required F_x, F_y.



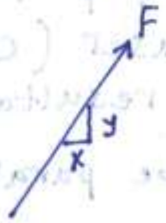


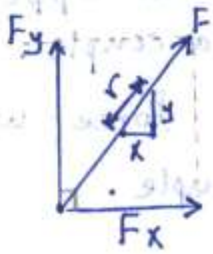
solution

$$F_x = F\cos\theta$$

$$F_y = F\sin\theta$$

b) Given F, slope of force (x, y). Req'd F_x, F_y.





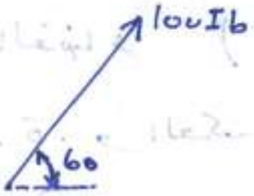
solution

$$r = \sqrt{x^2 + y^2}$$

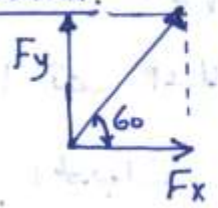
$$F_x = \frac{x}{r} F$$

$$F_y = \frac{y}{r} F$$

Ex Resolve the 100 Ib force [redacted]. into two components.



solution:



$$F_x = F\cos\theta$$

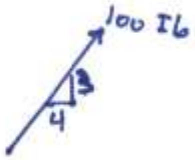
$$= 100\cos 60$$

$$= 50 \text{ Ib} \rightarrow$$

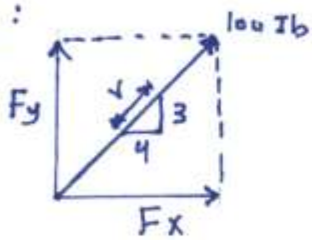
$$F_y = F\sin\theta = 100\sin 60$$

$$= 86.6 \text{ Ib} \uparrow$$

Ex Resolve the 100 Ib force into two components.



solution:



$$r = \sqrt{4^2 + 3^2} = 5$$

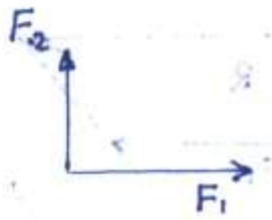
$$F_x = \frac{4}{5}(100) = 80 \text{ Ib} \rightarrow$$

$$F_y = \frac{3}{5}(100) = 60 \text{ Ib} \uparrow$$

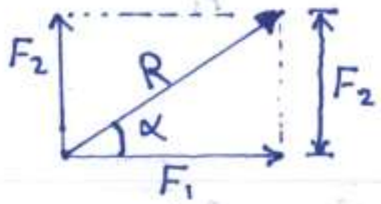
② Find resultant of two forces.

a) If the two forces are perpendicular.

Given F_1, F_2 . Req'd R, α



solution.

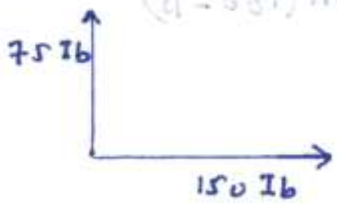


$$R = \sqrt{F_1^2 + F_2^2}$$

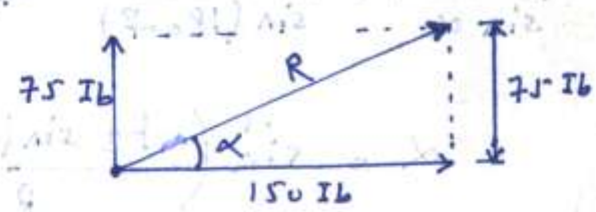
$$\tan\alpha = \frac{F_2}{F_1} \Rightarrow \alpha = \tan^{-1}\left(\frac{F_2}{F_1}\right)$$

$\therefore$

EX: Find resultant of two perpendicular forces.

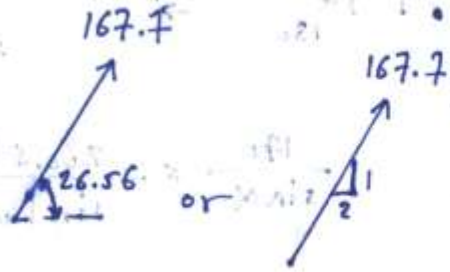


solution:



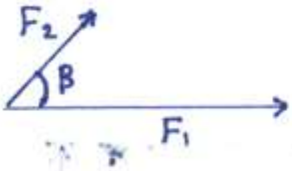
$$R = \sqrt{150^2 + 75^2} = 167.7 \text{ Ib}$$

$$\tan\alpha = \frac{75}{150} \Rightarrow \alpha = 26.56^\circ$$

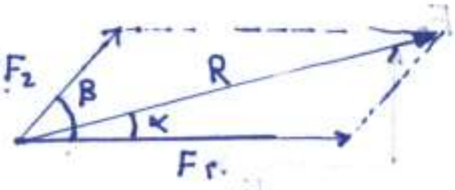


b) If the two forces making angle (acute or obtuse).

Given F_1, F_2, β. Req'd R, α.

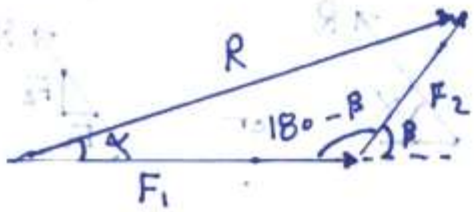


<u>solution</u>



$$R = \sqrt{F_1^2 + F_2^2 + 2F_1 F_2 \cos\beta}$$

to Find α use triangular law



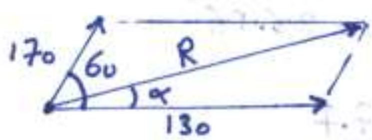
apply sine law

$$\frac{F_2}{\sin\alpha} = \frac{R}{\sin(180-\beta)} \Rightarrow R\sin\alpha = F_2 \sin(180-\beta)$$

$$\alpha = \sin^{-1}\left(\frac{F_2 \sin(180-\beta)}{R}\right)$$

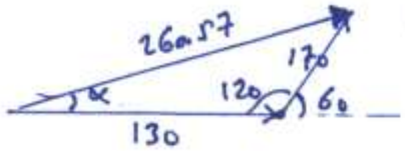
<u>EX:</u> Find resultant of two forces.

<u>solution</u>

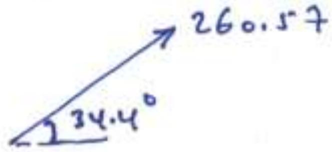


$$R = \sqrt{130^2 + 170^2 + 2(130)(170)\cos 60}$$

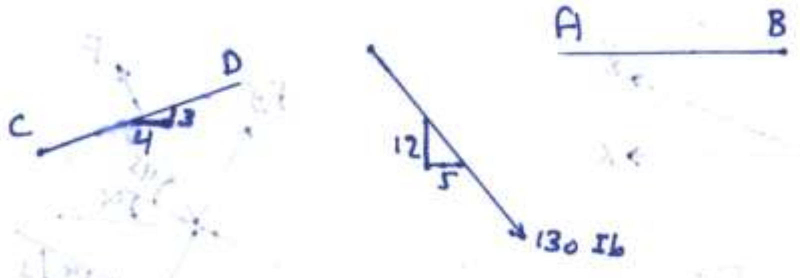
$$= 260.57 \text{ Ib}$$



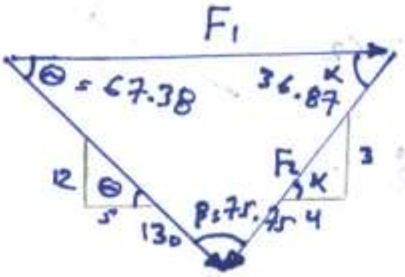
$$\frac{170}{\sin\alpha} = \frac{260.57}{\sin 120} \Rightarrow \underline{\alpha = 34.4^\circ}$$



1-9: Resolve the 130 Ib force into two nonrectangular components, on[illegible] a line of action along AB and the other parallel to CD.



Sol.



$\tan\theta = \frac{12}{5}$

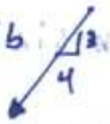
$\therefore \theta = 67.38^\circ$

$B = 180 - \alpha - \theta = 75.75$

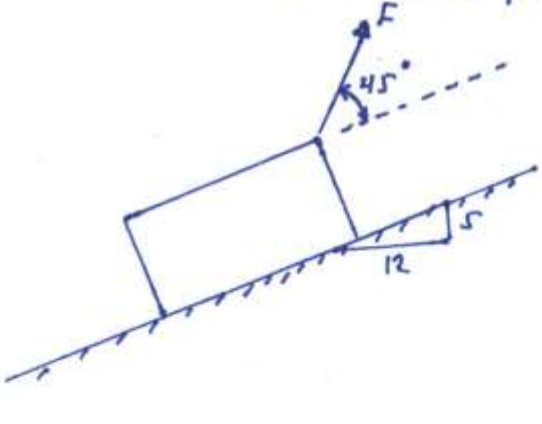
$\tan\alpha = \frac{3}{4} \rightarrow \alpha = 36.87^\circ$

$$\frac{F_1}{\sin 75.75} = \frac{130}{\sin 36.87} \Rightarrow F_1 = 210 \text{ Ib} \rightarrow \text{ along } AB$$

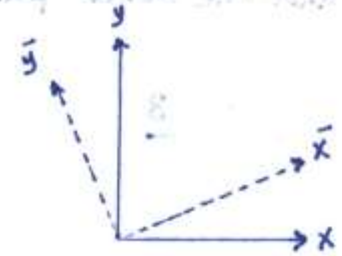
$$\frac{F_2}{\sin 67.38} = \frac{130}{\sin 36.87} \Rightarrow F_2 = 200 \text{ Ib} \text{ parallel to } CD$$

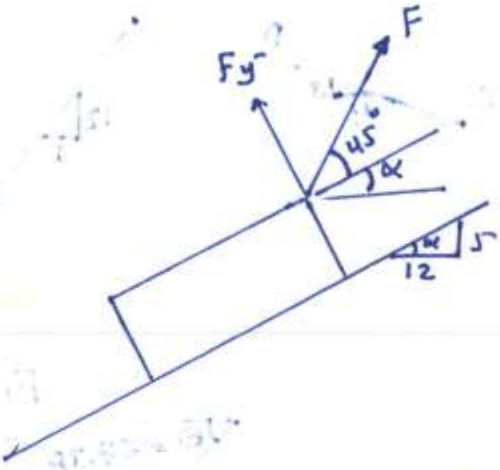


1-12 Q: The force F which acts on the block has a horizontal rectangular component of 100 Ib. Determine the rectangular component of F that is perpendicular to the inclined plane.



solution

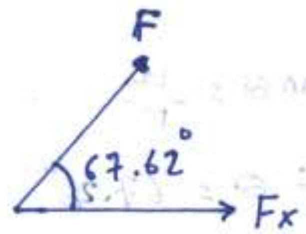




$F_x = 100\ \text{Ib} \qquad F_{\bar{y}} = ?$

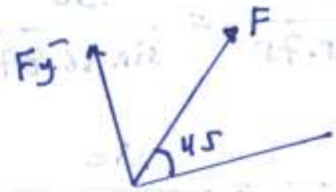
$\tan\alpha = \frac{5}{12} \Rightarrow \alpha = 22.62$

$45 + 22.62 = 67.62^\circ$



$F_x = F \cos 67.62$

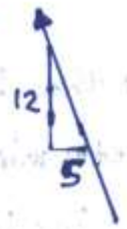
$100 = F \cos 67.62 \Rightarrow F = 262.64\ \text{Ib}$



$F_{\bar{y}} = F \sin 45$

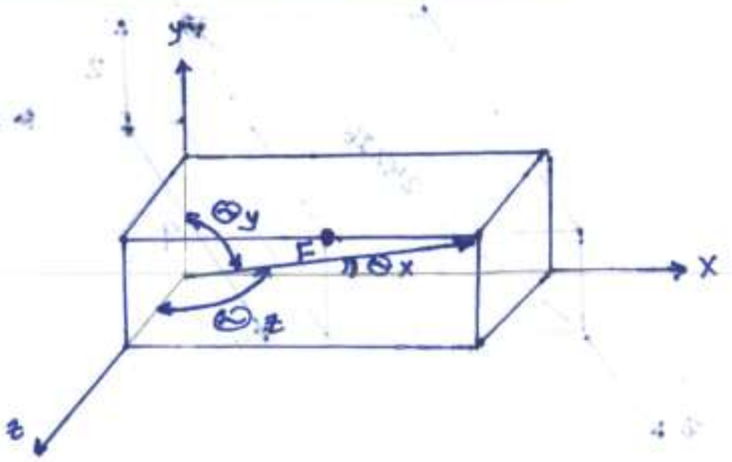
$= 262.64 * \sin 45$

$= 185.7\ \text{Ib}$



In space

① Resolve a force (in space) into three rectangular components.



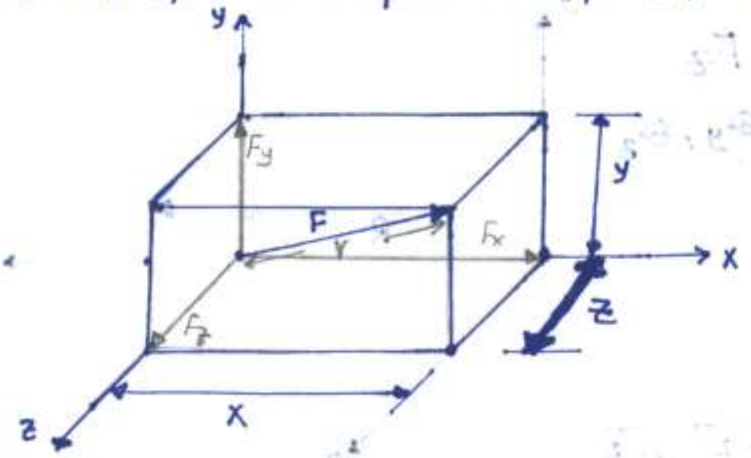
a) Given $F, \theta_x, \theta_y, \theta_z$. Req'd F_x, F_y, F_z

$$F_x = F \cos \theta_x$$

$$F_y = F \cos \theta_y$$

$$F_z = F \cos \theta_z$$

b) Given F, x, y, z. Req'd F_x, F_y, F_x



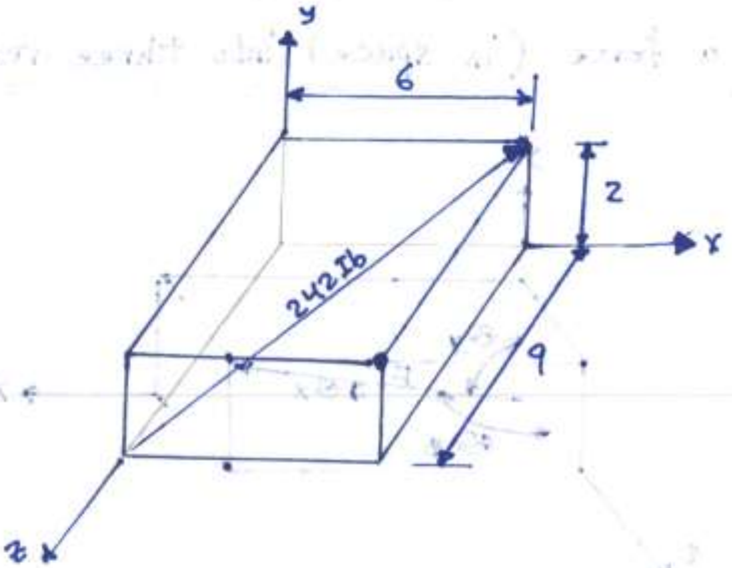
sol.

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$F_x = \frac{x}{r} F \qquad F_z = \frac{z}{r} F$$

$$F_y = \frac{y}{r} F$$

Q: Determine a set of three rectangular components of the 242 Ib force.



Sol. $r = \sqrt{6^2 + 2^2 + 9^2} = 11$

$F_x = \frac{6}{11}(242) = 132$ Ib $\rightarrow$

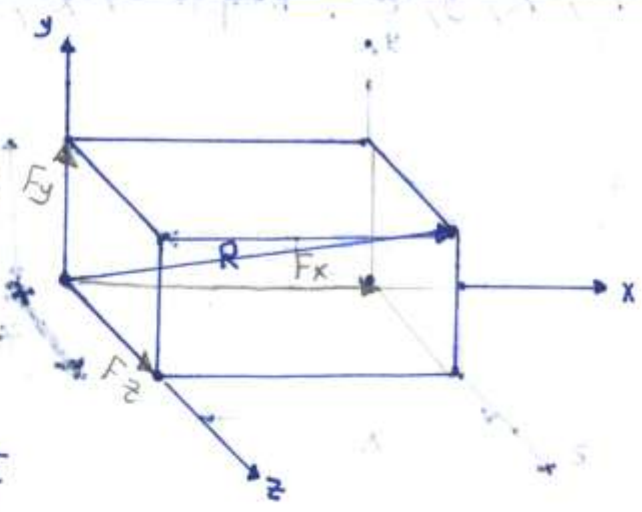
$F_y = \frac{2}{11}(242) = 44$ Ib $\uparrow$

$F_z = \frac{9}{11}(242) = 198$ Ib $\nearrow$

② Find resultant of three perpendicular forces.

Given F_x, F_y, F_z

Req'd $R, \theta_x, \theta_y, \theta_z$



Sol.

$R = \sqrt{F_x^2 + F_y^2 + F_z^2}$

$\cos\theta_x = \frac{F_x}{R}$

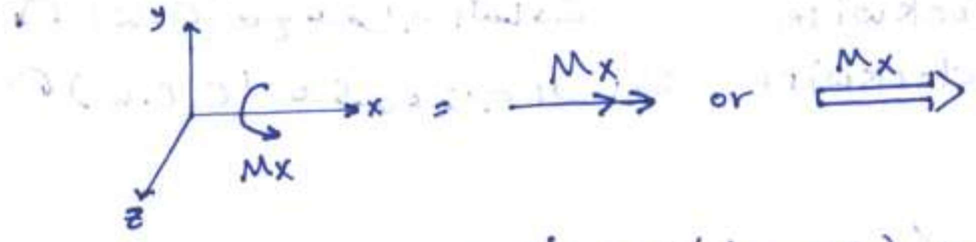
$\cos\theta_y = \frac{F_y}{R}$

$\cos\theta_z = \frac{F_z}{R}$

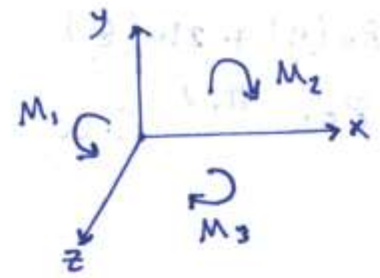
moment:

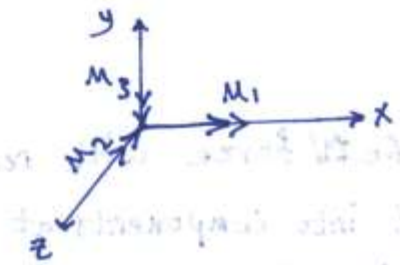
Moment (M) = Force * distance (مسافة عمودية على القوة)

يمثل العزم احياناً بسهم ذو رأسين ونستخدم طريقة لف اليد اليمنى لذلك.

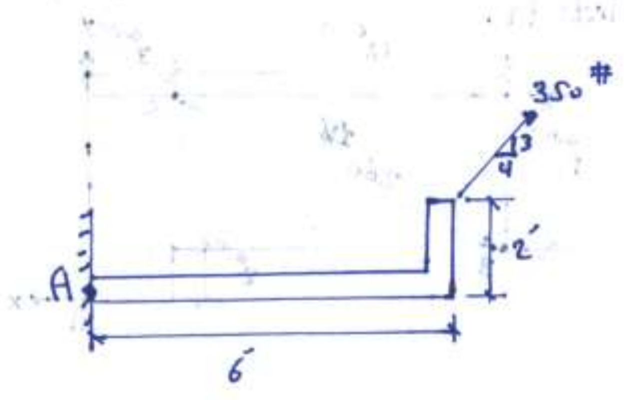


العزم (M_1) الموجود في الـ (plane yz) يكون تأثيره على (x-axis) والعزم (M_2) الموجود في الـ (plane xy) يكون تأثيره على (z-axis) والعزم (M_3) الموجود في الـ (plane xz) يكون تأثيره على (y-axis).

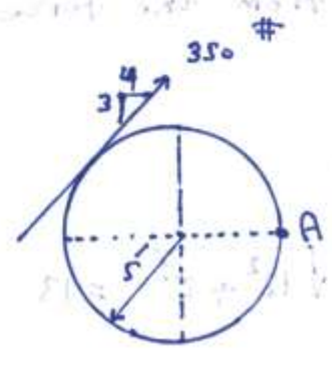




Q: Find the moment of the 350 Ib force with respect to point A.



(a)



(b)

solution

(a) Force 350#

$F_x = \frac{4}{5}(350) = 280^{\#} \rightarrow$

$F_y = \frac{3}{5}(350) = 210^{\#} \uparrow$

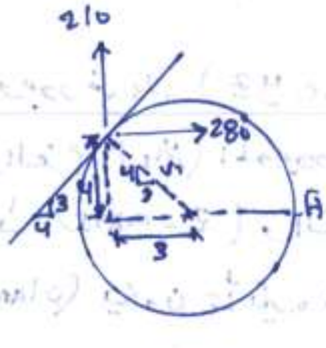
$M_A = 280(2) - 210(6) = -700$

$\therefore M_A = 700$ #.' (C.C.W)

Clockwise مع عقرب الساعة (C.W)

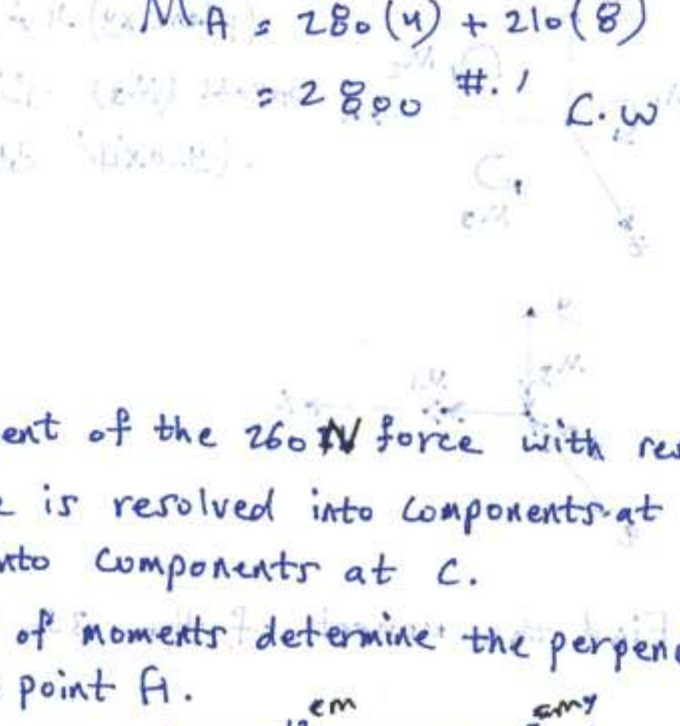
Counter clockwise عكس عقرب الساعة (C.C.W)

(b)



$M_A = 280(4) + 210(8)$
$= 2800$ #.' C.W

Q: (a) Determine the moment of the 260 N force with respect to point A when (1) the force is resolved into components at B; (2) the force is resolved into components at C.
(b) By means of the principle of moments determine the perpendicular distance from the force to point A.



solution

(a)

$r_{260} = \sqrt{12^2 + 5^2} = 13$

$F_x = \frac{12}{13}(260) = 240$ # ←

$F_y = \frac{5}{13}(260) = 100$ # ↓

نحلل القوة 260 عند النقطة B

$M_A = 100(15) + 240(2) = 1980$ #." C.C.W

نحلل القوة 260 عند النقطة C

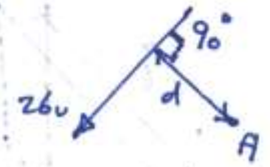
$M_A = 100(3) + 240(7) = 1980$ #." C.C.W

∴ يمكن تحليل القوة الى مركباتها في اي نقطة تقع على القوة أو امتدادها .

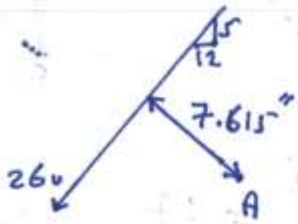
(15)

مجموع عزوم المركبات حول تلك النقطة = عزم المحصلة حول نقطة معينة

$$M_R)_A = \Sigma M_{force})_A$$

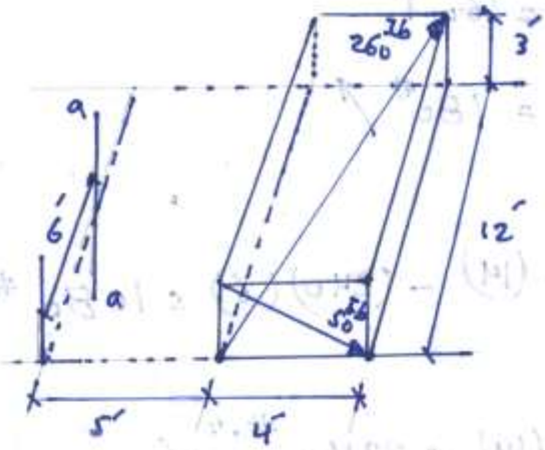


$$260 * d = 1980 \rightarrow d = 7.615''$$



Q: Find moment about line a-a.

(1-36)



sol.

Force 50#.

$$r_{50} = \sqrt{4^2+3^2} = 5$$

$$F_x = \frac{4}{5} * 50 = 40^{\#} \rightarrow$$

$$F_y = \frac{3}{5} * 50 = 30^{\#} \downarrow$$

Force 260#.

$$r_{260} = \sqrt{4^2+3^2+12^2} = 13$$

$$F_x = \frac{4}{13} * 260 = 80^{\#} \rightarrow$$

$$F_y = \frac{3}{13} * 260 = 60^{\#} \uparrow$$

$$F_z = \frac{12}{13} * 260 = 240^{\#} \nearrow$$

$$\overset{+}{\circlearrowleft} M_{@\ line\ a-a} = 240(5) + \overset{(40+80)}{(120)}(6)$$

$$= 1920^{\#}.'$$

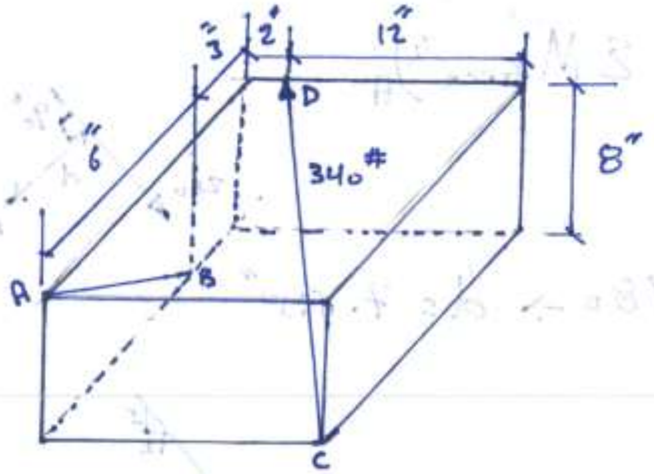
ملاحظة مهمة:

1) القوة التي توازي محور لا تعمل عزماً حوله

2) القوة التي تقطع محور لا تعمل عزماً حوله.

Q: A 340# force acts along the line from C to D. Determine the moment of this force with respect to the line AB.

(1-37)



Sol.

$$r_{340} = \sqrt{12^2 + 8^2 + 9^2} = 17$$

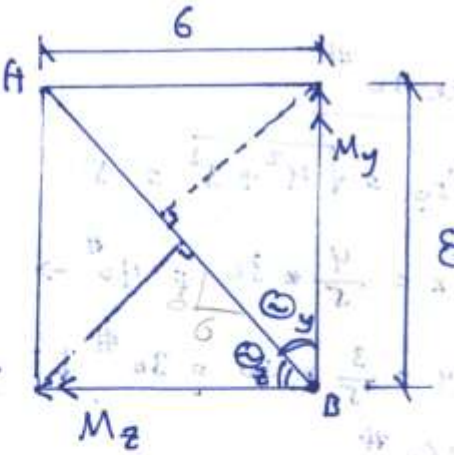
$$F_x = \frac{12}{17} * 340 = 240^{\#} \leftarrow$$

$$F_y = \frac{8}{17} * 340 = 160^{\#} \uparrow$$

$$F_z = \frac{9}{17} * 340 = 180^{\#} \nearrow$$

$$+\circlearrowleft M_{\substack{y\text{-axis} \\ \text{at } B}} = 180(14) - (240)(6) = 1080 \ \#.'' \uparrow$$

$$\circlearrowleft M_{\substack{z\text{-axis} \\ \text{at } B}} = 160(14) = 2240 \ \#.'' \swarrow$$

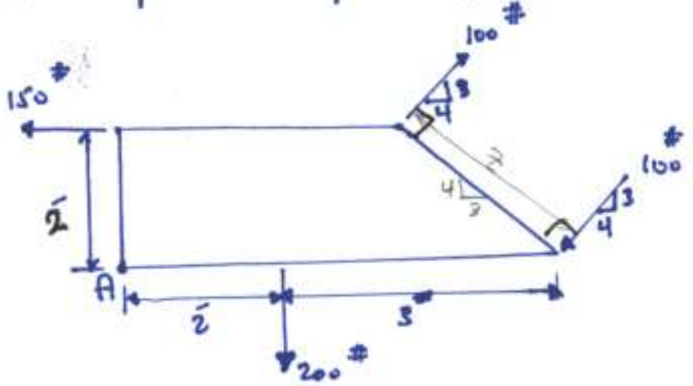


$$M_{@ \text{ line } AB} = M_y \cos\theta_y + M_z \cos\theta_z$$

$$= 1080 * \frac{8}{10} + 2240 * \frac{6}{10}$$

$$= 2208 \ \#.''$$

Q: Replace the force system with a single force. locate the force with respect to point A.



Sol.

$$\frac{z}{5} = \frac{2}{4} \rightarrow z = 2.5'$$

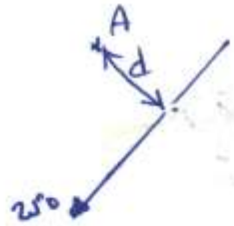
$$\text{couple} = 100(2.5) = 250 \ \#.'$$

$$R_x = \Sigma F_x = 150^{\#} \leftarrow$$

$$R_y = \Sigma F_y = 200^{\#} \downarrow$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(150)^2 + (200)^2} = 250$$

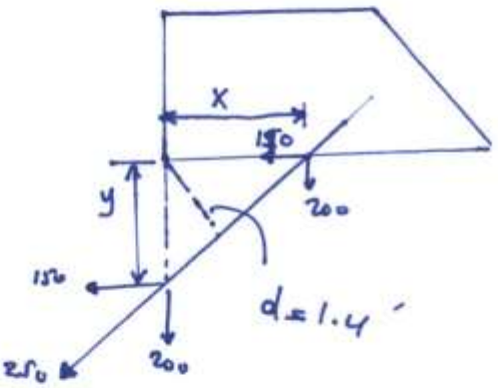
$$M_R)_A = \Sigma M_{force})_A$$



$$250 * d = -150(2) + 200(2) + 250$$

$$d = +1.4'$$

كيف يتم ايجاد x, y



$$250(1.4) = 150(y)$$

$$\rightarrow y = 2.333'$$

$$250(1.4) = 200(x)$$

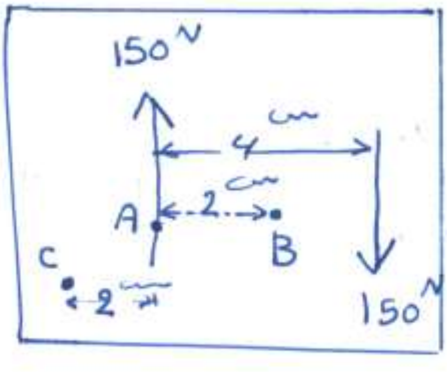
$$\rightarrow x = 1.75'$$

Couples

A couple consists of two forces which have equal magnitudes and parallel noncollinear lines of action but which are opposite in sense.

Q By using
Determine the moment of the couple with respect to point A, B & C

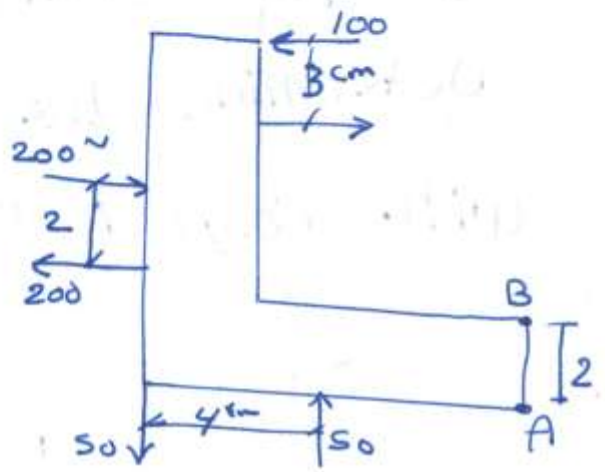
Moment at A, B & C equal



$M = 150 * 4$

$= 600 \ N \cdot cm$

Q: By using the transformations of a couple, replace the three couples by one couple with the forces acting horizontally at A & B.



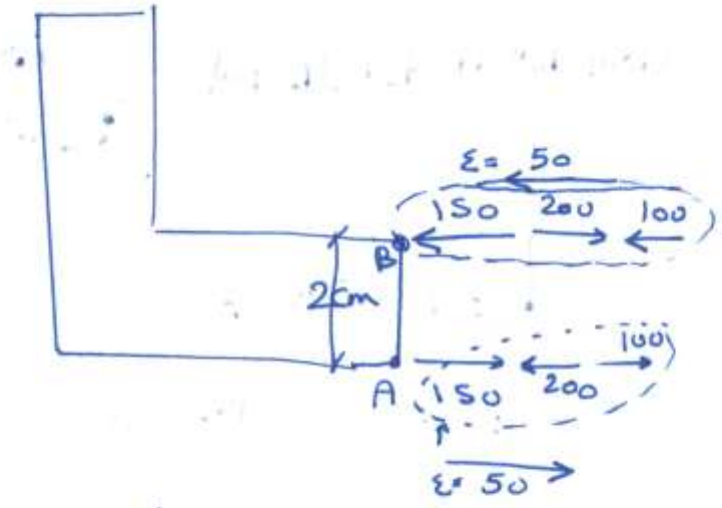
$M_1 = 100 * 3 = 300\ N\cdot cm$

$F_1 = \frac{300}{2} = 150\ N$

$F_2 = 200$

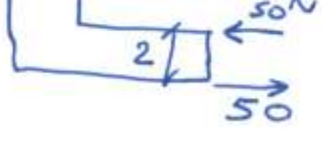
$M_3 = 50 * 4 = 200\ N\cdot cm$

$F_3 = \frac{200}{2} = 100\ N\cdot cm$

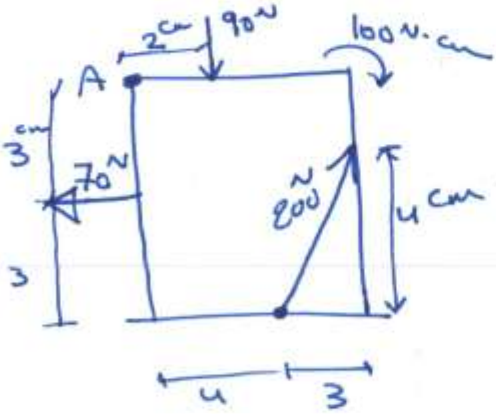


$F_A = 150 - 200 + 100$

$F_A = 50 \rightarrow$



Q Determine the resultant of the force system and locate it with respect to point A.



Sol

$F_x = 200\,\frac{3}{5} = 120^N \rightarrow$

$F_y = 200\,\frac{4}{5} = 160^N \uparrow$

$\overset{+}{\rightarrow}$ $R_x = \Sigma F_x = 120 - 70 = 50^N \rightarrow$

$+\uparrow R_y = \Sigma F_y = 160 - 90 = 70^N \uparrow$

$R = \sqrt{50^2 + 70^2} = 86\,N$

$\overset{+}{\circlearrowright} M_R)_A = \Sigma M$ force @ A

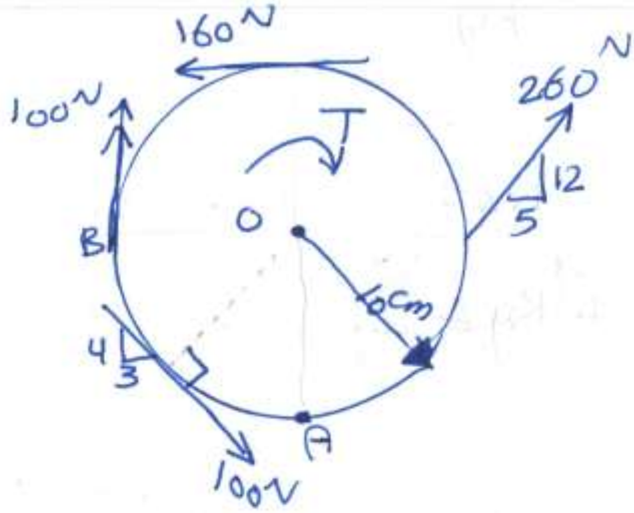
$86 * d = 90(2) + 70(3) + 100 - 120(6) - 160(4)$

$d = 10.11$

Q/

(22)

The resultant of the three forces & the couple T & an unknown force through point A is the vertical 100 N force through point B. Determine the unknown force through A & the magnitude of the couple T.



Sol

resultant 100 N

$R_x = 0$, $R_y = 100^N \uparrow$

Force 260 N

$r_{260} = \sqrt{5^2+12^2} = 13$

$F_x = 260\left(\frac{5}{13}\right) = 100^N \rightarrow$

$F_y = 260\left(\frac{12}{13}\right) = 240^N \uparrow$

Force 100 N

$r_{100} = \sqrt{3^2+4^2} = 5$

$F_x = 100 * \frac{3}{5} = 60\,N \rightarrow$

$F_y = 100\,\frac{4}{5} = 80^N \downarrow$

$R_x = \Sigma F_x \xrightarrow{+}$

$0 = 100 + 60 - 160 + F_x \rightarrow \boxed{F_x = 0}$

$R_y = \Sigma F_y \uparrow +$

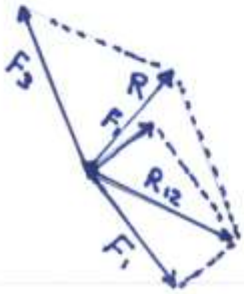
$100 = 240 - 80 + F_y \longrightarrow F_y = -60 \Rightarrow \boxed{F_y = 60^N \downarrow \text{ at } A}$

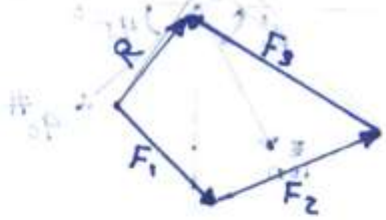
$+\circlearrowleft\; MR)_o = \Sigma M\,forces)_o$

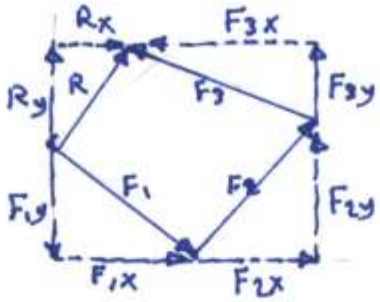
$100 * 10 = -160 * 10 - 100 * 10 - 240\,(10) + T \rightarrow \boxed{T = 6000^{N.cm}}$

Resultants of Force Systems.

Resultant of a concurrent, coplanar Force system.







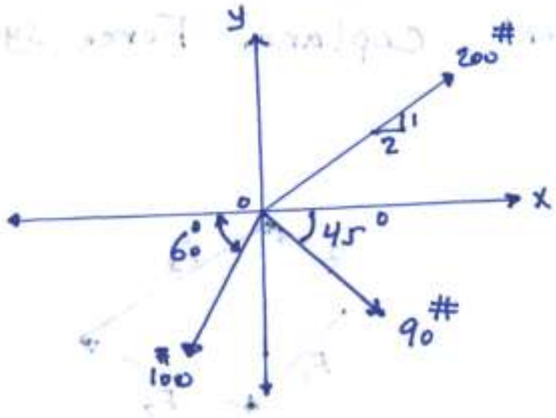
$$R_x = \Sigma F_x$$

$$R_y = \Sigma F_y$$

$$R = \sqrt{R_x^2 + R_y^2}$$

$$\tan \theta_x = \frac{R_y}{R_x}$$

Ex: Determine the resultant.



sol.

Force 100#:

$F_x = 100 \cos 60 = 50^{\#} \leftarrow$

$F_y = 100 \sin 60 = 86.6^{\#} \downarrow$

Force 90#,

$F_x = 90 \cos 45 = 63.64^{\#} \rightarrow$

$F_y = 90 \sin 45 = 63.64^{\#} \downarrow$

Force 200#:

$r = \sqrt{2^2 + 1^2} = \sqrt{5}$

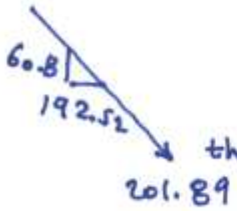
$F_x = \frac{2}{\sqrt{5}} (200) = 178.88^{\#} \rightarrow$

$F_y = \frac{1}{\sqrt{5}} (200) = 89.44^{\#} \uparrow$

$\overset{+}{\rightarrow} R_x = \Sigma F_x = -50 + 63.64 + 178.88 = 192.52^{\#} \rightarrow$

$+\downarrow R_y = \Sigma F_y = 86.6 + 63.64 - 89.44 = 60.8^{\#} \downarrow$

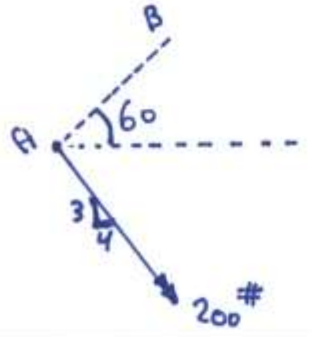
$R = \sqrt{R_x^2 + R_y^2} = \sqrt{192.52^2 + 60.8^2} = 201.89$

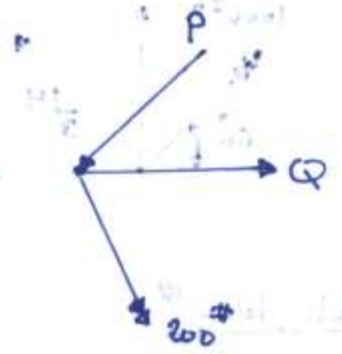


through o

Q: The 200 lb force is the resultant of two forces. One of the forces, P, has its direction along the line AB, and the other force, Q, is known to be in the horizontal direction. Determine the forces P and Q.

(11)





sol.

Resultant $200^{\#}$.

$R_x = 200\left(\frac{4}{5}\right) = 160^{\#} \rightarrow$

$R_y = 200\left(\frac{3}{5}\right) = 120^{\#} \downarrow$

Force P.

$F_x = P\cos 60 = 0.5P \leftarrow$

$F_y = P\sin 60 = 0.866P \downarrow$

Force Q

$F_x = Q \rightarrow$

$\overset{+}{\rightarrow} R_x = \Sigma F_x$

$\boxed{160 = Q - 0.5P}$ —— (1)

$+\downarrow R_y = \Sigma F_y$

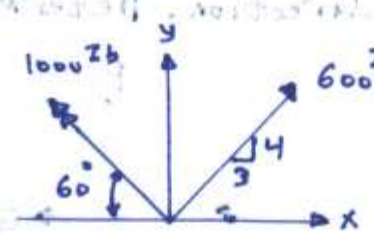
$120 = 0.866P \Rightarrow P = 138.56^{\#}$ 60 through A

نعوض بمعادلة (1)

$160 = Q - 0.5(138.56)$

$\rightarrow Q = 229.28^{\#} \longrightarrow$ through A

Ex! The 1000 Ib force is the resultant of two forces, one of which is 600 Ib. Determine the other force.



sol.

resultant $1000^{\#}$

$R_x = 1000 \cos 60 = 500^{\#} \leftarrow$

$R_y = 1000 \sin 60 = 866^{\#} \uparrow$

force $600^{\#}$

$F_x = 600\left(\frac{3}{5}\right) = 360^{\#} \rightarrow$

$F_y = 600\left(\frac{4}{5}\right) = 480^{\#} \uparrow$

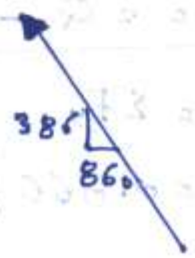
$\overset{+}{\leftarrow} R_x = \Sigma F_x$

$500 = -360 + F_x \rightarrow F_x = 860^{\#} \leftarrow$

$+\uparrow R_y = \Sigma F_y$

$866 = 480 + F_y \rightarrow F_y = 386^{\#} \uparrow$

$R = \sqrt{F_x^2 + F_y^2} = \sqrt{860^2 + 386^2} = 942.65^{\#}$

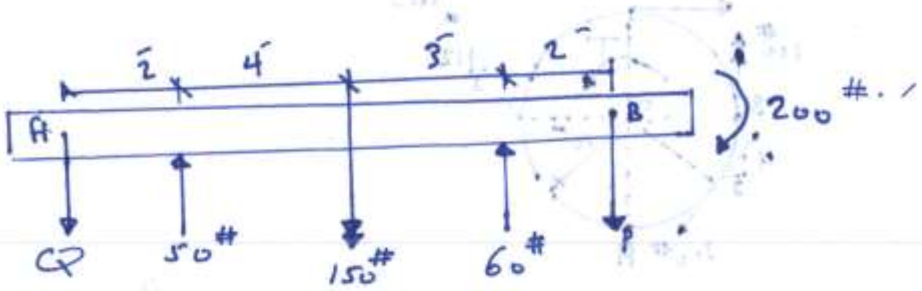


through origin

Resultant of a Noncurrent, Coplanar force system.

(1P)

Q: The 150# force is the resultant of the four forces shown and the couple. Determine the forces P and Q.



sol.

$+\uparrow R_y = \Sigma F_y$

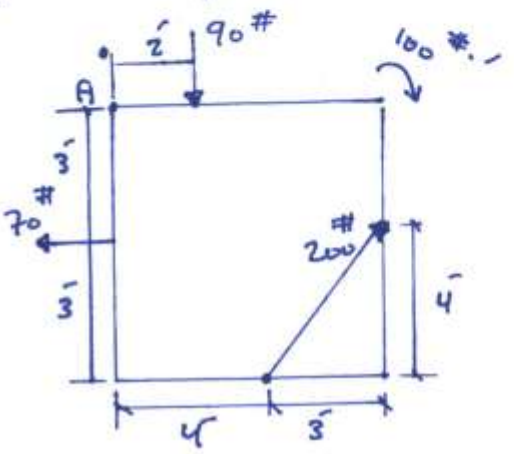
$-150 = -Q - P + 50 + 60 \rightarrow \boxed{Q + P = 260}$ —— (1)

$\curvearrowleft + \; M_R)_A = \Sigma M_{force})_A$

$150(6) = -50(2) - 60(9) + P(11) + 200 \rightarrow P = 121.82^{\#} \downarrow$

$\therefore Q = 260 - 121.82 = 138.18^{\#} \downarrow$

Q: Determine the resultant of the force system, and locate it with respect to point A.



sol.

Force 200#

$F_x = 200\left(\frac{3}{5}\right) = 120^{\#} \rightarrow$

$F_y = 200\left(\frac{4}{5}\right) = 160^{\#} \uparrow$

Force 90#

$F_y = 90^{\#} \downarrow$

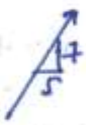
Force 70#

$F_x = 70^{\#} \leftarrow$

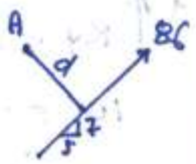
$\xrightarrow{+} R_x = \Sigma F_x = 120 - 70 = 50^{\#} \rightarrow$

$+\uparrow R_y = \Sigma F_y = 160 - 90 = 70^{\#} \uparrow$

$R = \sqrt{50^2 + 70^2} = 86^{\#}$



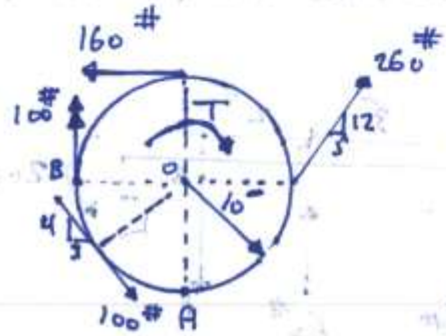
$\curvearrowleft + \; M_R)_A = \Sigma M_{force})_A$



$86 * d = +90(2) + 70(3) + 100 + 120(6) + 160(4)$

$\therefore d = 10.11'$

Q: The resultant of the three forces and the couple T and an unknown force through point A is the vertical 100 lb force through point B. Determine the unknown force through A and the magnitude of the couple T.



sol.

resultant 100#

$R_x = 0$, $R_y = 100^{\#} \uparrow$

force 100#

$F_x = 100\left(\frac{3}{5}\right) = 60^{\#} \rightarrow$

$F_y = 100\left(\frac{4}{5}\right) = 80^{\#} \downarrow$

force 260#

$r_{260} = \sqrt{5^2+12^2} = 13$

$F_x = 260\left(\frac{5}{13}\right) = 100^{\#} \rightarrow$

$F_y = 260\left(\frac{12}{13}\right) = 240^{\#} \uparrow$

Force 160#

$F_x = 160^{\#} \leftarrow$

$\overset{+}{\rightarrow} R_x = \Sigma F_x$

$0 = 60 + 100 - 160 + F_x$

$\boxed{F_x = 0}$

$+\uparrow R_y = \Sigma F_y$

$100 = -80 + 240 + F_y \rightarrow F_y = -60$

$\therefore \boxed{F_y = 60^{\#} \downarrow}$

$\therefore R_A = 60^{\#} \downarrow$

T ايجاد

$\overset{+}{\curvearrowleft} \left. M_R \right)_O = \Sigma \left. M_{force} \right)_O$

$100(10) = \underbrace{-60(6) - 80(8) - 240(10)}_{-100 \times 10} - 160(10) + T$

$T = 6000^{\#}$

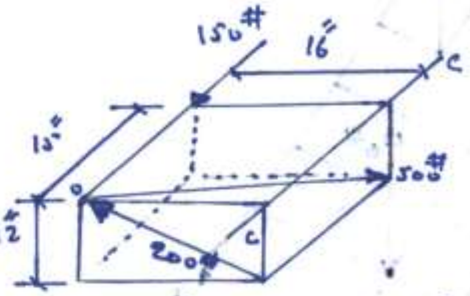
Resultant of a concurrent, Noncoplanar force system.

$$R_x = \Sigma F_x \; ; \; R_y = \Sigma F_y \; ; \; R_z = \Sigma F_z$$

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

$$\cos\theta_x = \frac{R_x}{R} \; ; \; \cos\theta_y = \frac{R_y}{R} \; ; \; \cos\theta_z = \frac{R_z}{R}$$

Q: Determine the resultant of the force system and its moment with respect to the C axis.



sol. Force 500#:

$$r_{500} = \sqrt{16^2 + 12^2 + 15^2} = 25$$

$$F_x = \frac{16}{25}(500) = 320^{\#} \rightarrow$$

$$F_y = \frac{12}{25}(500) = 240^{\#} \downarrow$$

$$F_z = \frac{15}{25}(500) = 300^{\#} \nearrow$$

Force 200#:

$$r_{200} = \sqrt{16^2 + 12^2} = 20$$

$$F_x = \frac{16}{20}(200) = 160^{\#} \leftarrow$$

$$F_y = \frac{12}{20}(200) = 120^{\#} \uparrow$$

Force 150#

$$F_z = 150 \swarrow$$

$$\overset{+}{\rightarrow} R_x = \Sigma F_x = 320 - 160 = 160^{\#} \rightarrow$$

$$+\uparrow R_y = \Sigma F_y = -240 + 120 = -120$$

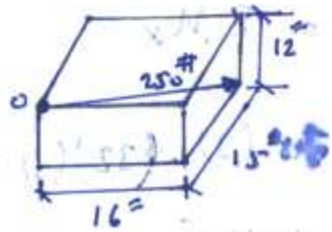
$$R_y = 120^{\#} \downarrow$$

$$+\nearrow R_z = \Sigma F_z = 300 - 150 = 150 \nearrow$$

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

$$= \sqrt{(160)^2 + (120)^2 + (150)^2}$$

$$= 250^{\#}$$

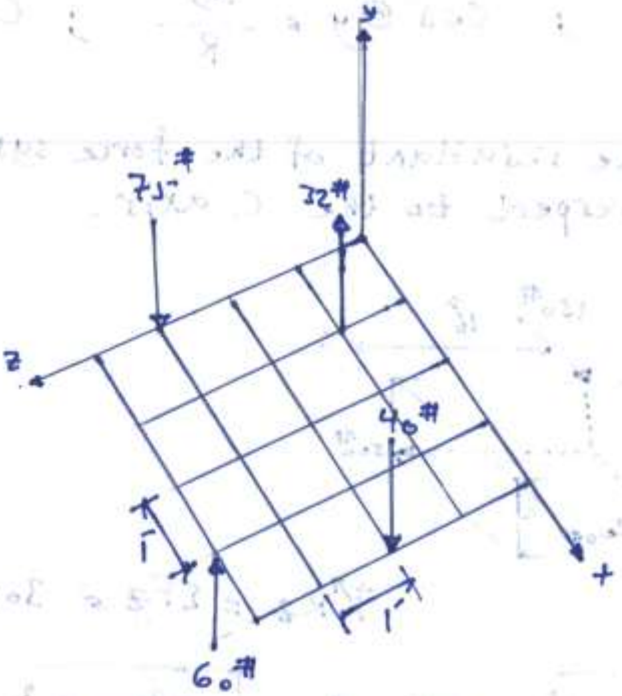


$$M_{@\,c\text{-axis}} = 120(16) = 1920^{\#.''}$$

Resultant of a parallel, Noncoplanar force system.

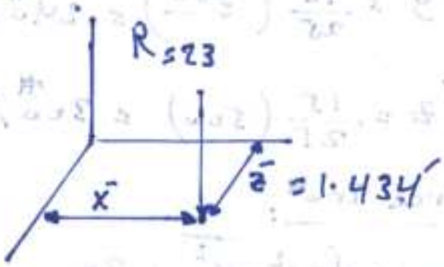
$$R = \Sigma F_y \quad ; \quad R\bar{x} = \Sigma M_z \quad ; \quad R\bar{z} = \Sigma M_x$$

Q: Determine the resultant of the four parallel forces and show it on a sketch.



$+\uparrow R = \Sigma F_y = 60 - 75 - 40 + 32 = -23$

$\therefore R = 23^{\#} \downarrow$



$R\bar{z} = \Sigma M_x$

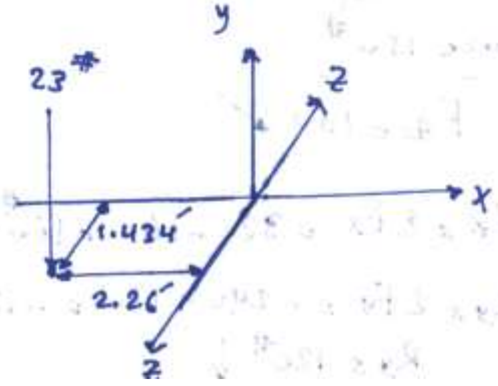
$23 * \bar{z} = -75(3) + (32)(1) - 40(2) + 60(4)$

$\rightarrow \bar{z} = -1.434'$

$+\curvearrowright R\bar{x} = \Sigma M_z$

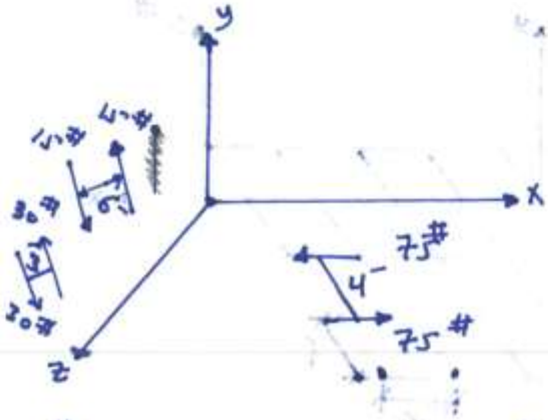
$23 * \bar{x} = -32(1) + 40(4) - 60(3)$

$\bar{x} = -2.26$



Resultant of a system of couples.

Q: Find the resultant of the system of couples shown in Fig.

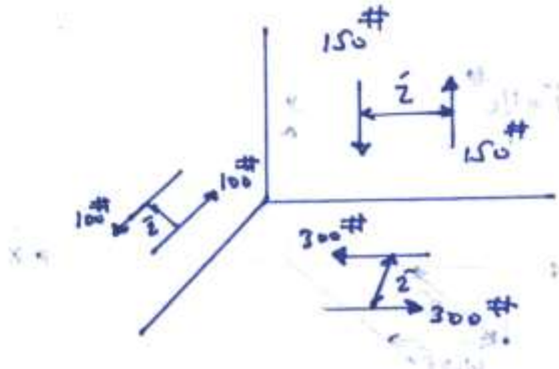


$\overset{+}{\curvearrowleft} M_x = 15(6) + 30(3) = 180^{\#\cdot\prime}$

$M_y = 75(4) = 300^{\#\cdot\prime}$

$M = \sqrt{M_x^2 + M_y^2} = \sqrt{180^2 + 300^2} = 349.86^{\#\cdot\prime}$

Q: Find the resultant of the system couples shown in Fig.



$\overset{+}{\curvearrowleft} M_x = 100(2) = 200^{\#\cdot\prime}$

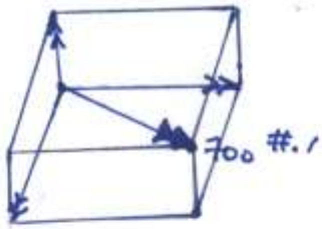
$M_y = 300(2) = 600^{\#\cdot\prime}$

$M_z = 150(2) = 300^{\#\cdot\prime}$

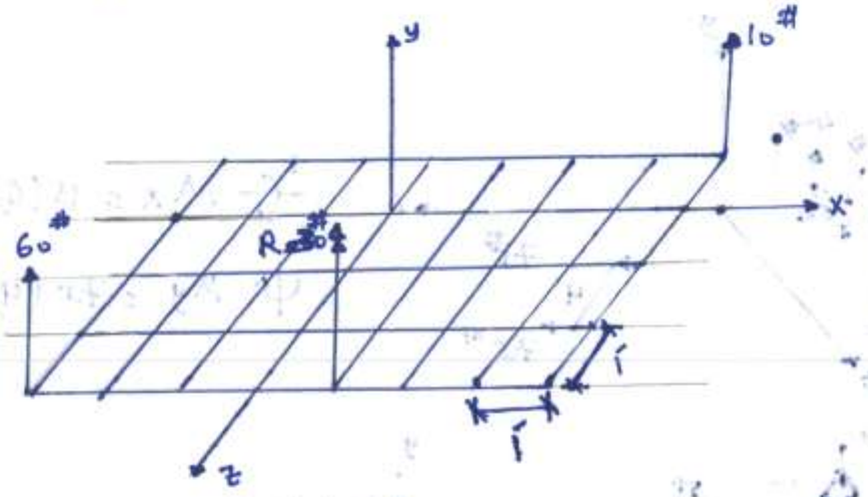
$M = \sqrt{M_x^2 + M_y^2 + M_z^2}$

$= \sqrt{200^2 + 600^2 + 300^2}$

$= 700^{\#\cdot\prime}$



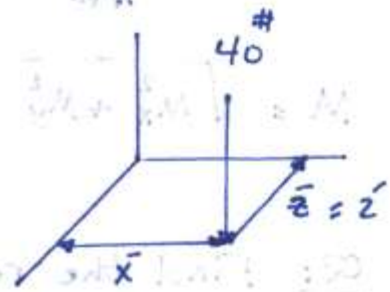
Q: In the parallel force system, the 30Ib force is the resultant of three forces, two of which are shown. Determine the third force, and locate it on a sketch. (28)



sol.

$+\uparrow R = \Sigma F_y$

$30 = 60 + 10 + F \rightarrow F = -40 \quad \therefore F = 40^{\#} \downarrow$



$R\bar{z} = \Sigma M_x$

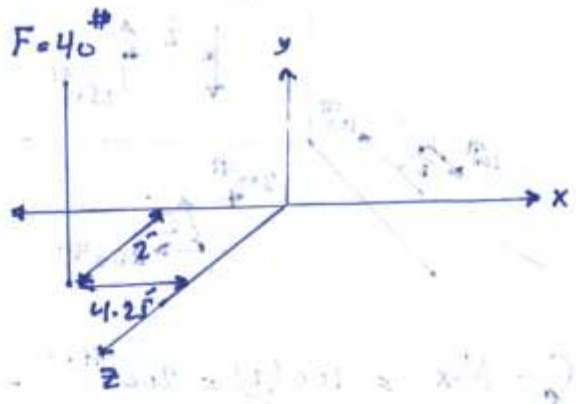
$+30(3) = 60(3) - 10(1) + 40 * \bar{z}$

$\bar{z} = 2$

$R\bar{x} = \Sigma M_z$

$-30(1) = -10(4) + 60(3) + 40 * \bar{x}$

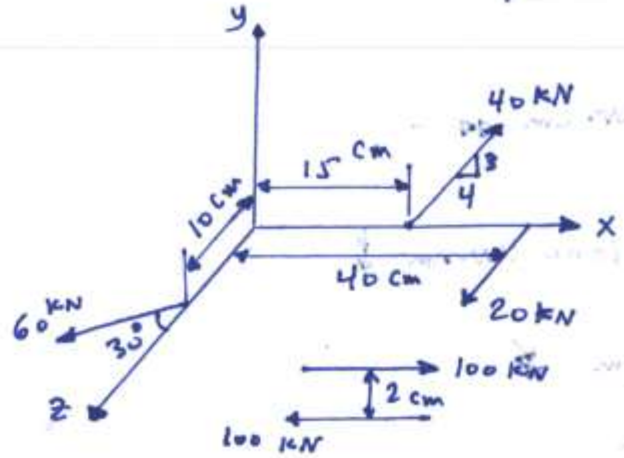
$\bar{x} = -4.25$



Resultant of a Nonconcurrent, Nonparallel, Noncoplanar force system.

The resultant of a nonconcurrent, parallel, noncoplanar force system can be a single or couple, but in general it is a force and a couple.

Q: Determine the resultant of the general force system.



sol.

Force 60^{kN}:

$F_y = 60 \sin 30 = 30^{kN} \uparrow$

$F_z = 60 \cos 30 = 51.96^{kN} \swarrow$

Force 40^{kN}:

$F_x = 40\left(\frac{4}{5}\right) = 32^{kN} \rightarrow$

$F_y = 40\left(\frac{3}{5}\right) = 24^{kN} \uparrow$

Force 20^{kN}:

$F_z = 20^{kN} \swarrow$

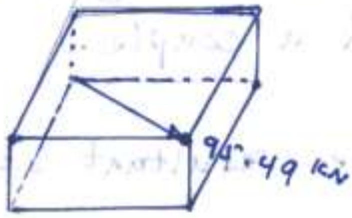
$\xrightarrow{+} R_x = \Sigma F_x = 32^{kN} \rightarrow$

$+\uparrow R_y = \Sigma F_y = 30 + 24 = 54\ kN \uparrow$

$+\swarrow R_z = \Sigma F_z = 51.96 + 20 = 71.96\ kN \swarrow$

(31)

$\therefore\ R = \sqrt{32^2 + 54^2 + 71.96^2} = 95.49\ \text{kN}$

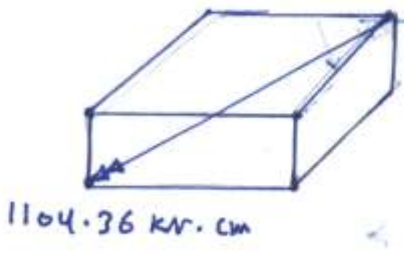


$\overset{+}{\curvearrowleft} M_x = 30(10) = 300\ \text{kN.cm}$

$\overset{+}{\curvearrowleft} M_y = 20(40) + 100(2) = 1000\ \text{kN.cm}$

$\overset{+}{\curvearrowleft} M_z = 24(15) = 360\ \text{kN.cm}$

$M = \sqrt{300^2 + 1000^2 + 360^2} = 1104.36\ \text{kN.cm}$



د. سرهد

Equilibrium

الجسم في حالة اتزان، اي ان المحصلة = صفر

In plane

$$\left.\begin{array}{l}\Sigma F_x = 0 \\ \Sigma F_y = 0 \\ \Sigma M = 0\end{array}\right\} \text{Equations of Equilibrium}$$

In Space

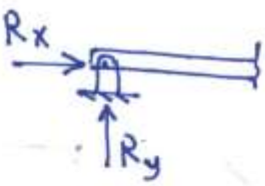
$$\left.\begin{array}{ll}\Sigma F_x = 0 & \Sigma M_{x\text{-}axis} = 0 \\ \Sigma F_y = 0 & \Sigma M_{y\text{-}axis} = 0 \\ \Sigma F_z = 0 & \Sigma M_{z\text{-}axis} = 0\end{array}\right\} \text{Equations of Equilibrium}$$

Reactions : ردود الافعال

Roller

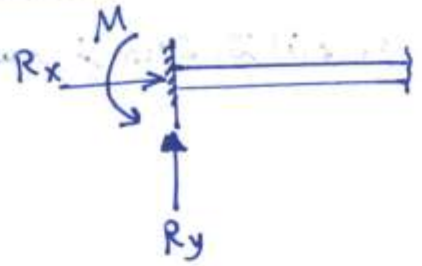
في حالة الـ roller رد الفعل يكون عمودي على السطح الذي تتحرك عليه الـ roller

Pin (Hinge) رد فعل عدد اثنين



في حالة الـ pin هنالك رد فعل عدد (2) Ry , Rx متعامدين

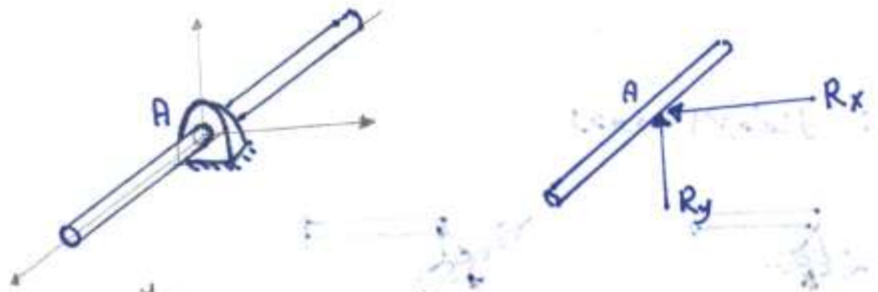
Fixed End رد فعل عدد ثلاثة

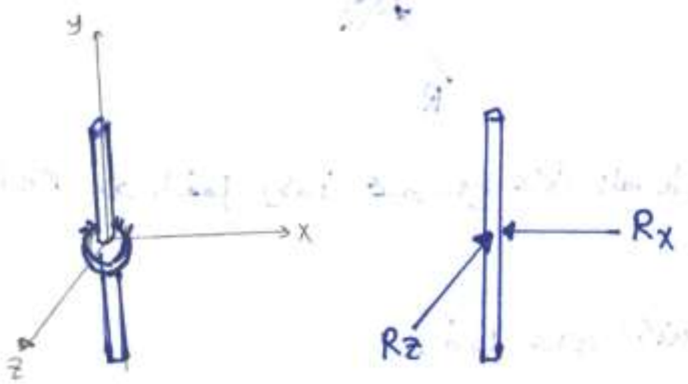


Cord, Cable

Cable ; Tensile force

smooth bearing





Ball and Socket. رد فعل عدد ثلاثة